Improved direct binary search-based algorithm for generating holograms for the application of holographic optical tweezers

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Abstract. This paper presents an improved direct binary search (DBS)-based algorithm for generating holograms to holographic optical tweezers. The simulations show that the improved algorithm greatly enhances computation speed while maintaining high hologram efficiency and high-intensity homogeneous target spots. The improved algorithm was applied to generate holographic optical tweezers in several experiments. The experiments demonstrate that real-time trap and manipulation can be realized with the improved algorithm if the number of trapped microparticles is small. © 2012 Society of Photo-Optical Instrumentation Engineers (SPIE). [DOI: 10.1117/1.OE.51.1.015801]

Subject terms: direct binary search; computer-generated holograms; spatial light modulator; holographic optical tweezers.

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1 Introduction
Holographic optical tweezers could flexibly trap and manipulate particles, which play an important role in many fields such as physics, chemistry, life sciences, micro- and nanomanufacturing, and microfluidics.1-5 The key to a holographic optical tweezers system is the hologram's design. So far, Gerchberg-Saxton (GS), DBS (Direct Binary Search), and other algorithms have been developed.6-9 These algorithms need to be optically efficient and fast enough for real-time manipulation.

DBS has excellent performance in terms of flexibility and output efficiency, but requires a great deal of computation time. The output plane phase optimize (OPPO) algorithm proposed by Georgiou et al.,10 is an alternative to DBS and very suitable for designing spot-generating holograms for the use in holographic optical tweezers. To create S-target spots on the output plane, the OPPO algorithm chooses randomly $4 - 5 S$ pixels from the hologram plane, and flip operations are only performed on the selected pixels in order to design a temporary hologram and form the target output plane (DBS is used in this step). Then, the phase of the output spots is measured, and the inverse DFT can be computed in order to obtain the full-resolution hologram.

However, the computing speed of OPPO is still a drawback. Based on this research, we improved this algorithm to reduce its run time. The experiments on the improved OPPO were also performed on the holographic optical tweezers platform.

2 Improved Algorithm
In the holographic optical tweezers system, the hologram plane and the output plane (i.e., the focal plane of the objective lens) are related by the Fourier transform. Based on the OPPO algorithm mentioned in Ref. 10, some improvements were focused on temporary pixels and phase levels will be presented. The improved algorithm consists of five main steps for generating S-target spots on the output plane.

1. Randomly choose S-pixels from the hologram plane.
2. Using the selected S-pixels, design a 2-phase-level temporary hologram by DBS to optimize the output plane.
3. Obtain the phase data of the target spots on the output plane.
4. Use FFT together with the phase of the output spots to calculate the full-resolution hologram and quantize its phase values to the nearest $1/p$ ($p$ is the number of the SLM’s phase levels).
5. Compute the efficiency of the hologram: if it is less than the cut-off value (we set it to 80%), go to the first step and recalculate a new hologram.

It should be noted that in the second step of the improved algorithm, we use inverse fast Fourier transform (IFFT) in DBS to calculate the reconstruction (the output plane) and FFT in the fourth step to calculate the full-resolution hologram. This setting is the reverse of the original OPPO but the same as the classic DBS algorithm,6,7 which does not affect the algorithm’s effectiveness since the hologram plane and the output plane still keep their Fourier transform relationship.

3 Calculations
3.1 Effect of Temporary Phase Levels
During the optimization of the temporary hologram, random flip operations are performed on the phase of the pixels, and the effectiveness of the flips is observed through the cost function. When the cost function is minimized, the output spots’ phase is considered optimum.

In the second step of OPPO, the number of the temporary hologram’s phase levels is usually equal to the SLM’s. If the
Intensity mean square error

Efficiency 80.91% 80.91% 80.91% 80.91% 80.91% 80.91% 80.91% 80.91%

Run time(s) 35.63 18.03 9.28 5.06 2.89 1.88 1.36 1.16

Spot 4
Spot 3
Spot 2
Spot 1

ary hologram. The cost function presented in Ref. 10 was temporary pixels were chosen randomly to form the tempor-
same in every simulation.

also used in this improved algorithm. The positions of the
spots.

intensity mean square error (IMSE) of the four output
decreased from 256 to 2. The last row of the table gives the
ing that the phase levels of the temporary hologram
the program

be performed
s run time, and the hologram efficiency, show-
during the calculations, two

Here, the Matlab programming language was used to simulate the algorithm (calculations were performed on a computer with an Intel dual-Core 2.50 GHz processor) for a 4-spot-generating hologram with 256 × 256 pixels. Sixteen temporary pixels were chosen randomly to form the temporary hologram. The cost function presented in Ref. 10 was also used in this improved algorithm. The positions of the temporary pixels and the initial hologram were kept the same in every simulation.

Table 1 shows the phase of the 4 target spots (as radian), the program’s run time, and the hologram efficiency, showing that the phase levels of the temporary hologram decreased from 256 to 2. The last row of the table gives the intensity mean square error (IMSE) of the four output spots.

In Table 1, the hologram efficiency is defined as

\[ \eta = \frac{\sum_{i=1}^{S} p_i}{\sum_{j=1}^{MN} p_j}. \]

Table 1 Phase level and the phases of the output spots.

<table>
<thead>
<tr>
<th>Phase division</th>
<th>2x</th>
<th>2x</th>
<th>2x</th>
<th>2x</th>
<th>2x</th>
<th>2x</th>
<th>2x</th>
<th>2x</th>
</tr>
</thead>
<tbody>
<tr>
<td>256</td>
<td>0.3182</td>
<td>0.3184</td>
<td>0.3174</td>
<td>0.3186</td>
<td>0.3193</td>
<td>0.3193</td>
<td>0.3164</td>
<td>0.3237</td>
</tr>
<tr>
<td>128</td>
<td>2.5258</td>
<td>2.5270</td>
<td>2.5260</td>
<td>2.5251</td>
<td>2.5253</td>
<td>2.5275</td>
<td>2.5350</td>
<td>2.4319</td>
</tr>
<tr>
<td>64</td>
<td>-0.7367</td>
<td>-0.7351</td>
<td>-0.7358</td>
<td>-0.7379</td>
<td>-0.7404</td>
<td>-0.7253</td>
<td>-0.7571</td>
<td>-0.7663</td>
</tr>
<tr>
<td>32</td>
<td>-0.4613</td>
<td>-0.4612</td>
<td>-0.4613</td>
<td>-0.4591</td>
<td>-0.4624</td>
<td>-0.4633</td>
<td>-0.4619</td>
<td>-0.4740</td>
</tr>
<tr>
<td>16</td>
<td>35.63</td>
<td>18.03</td>
<td>9.28</td>
<td>5.06</td>
<td>2.89</td>
<td>1.88</td>
<td>1.36</td>
<td>1.16</td>
</tr>
<tr>
<td>8</td>
<td>80.91%</td>
<td>80.91%</td>
<td>80.91%</td>
<td>80.91%</td>
<td>80.91%</td>
<td>80.91%</td>
<td>80.91%</td>
<td>80.91%</td>
</tr>
</tbody>
</table>

where \( p_i \) is the power of the output spots, \( p_j \) is the power of the whole output plane, and \( M \times N \) is the total number of output plane pixels.

It can be seen that the optimum phase of the target spots obtained by the 2-phase-level hologram is different from the 256-phase-level hologram, but their efficiency and IMSE are almost the same. Meanwhile, the computation time decreased from 35.63 s to 1.16 s.

Furthermore, we computed hundreds of holograms in order to generate different numbers of spots with different positions, and the results still support our point.

It can be concluded that even with the simplest binary hologram, which has only two phase levels, the optimum phase on the output plane exists. So, it is unnecessary to set the phase levels of the temporary hologram to be equal to the SLM’s.

In addition, during hologram calculation, the pixels were only iterated once. Because the optimum phase of the output plane is decided after a single iteration, the results derived from additional iterations have no effect compared with those from the first iteration. The simulation results also support this claim. Taking the 2-phase-level hologram in Table 1, for instance, the output spots’ phase did not change after 2 to 5 iterations compared with the first iteration, as shown in Table 2. Therefore, during the calculations, two phase levels and one iteration were used.

### 3.2 Effect of the Temporary Pixels’ Number

On the basis of cutting down the number of the temporary phase levels, and in order to reduce the computing time further, the possibility of reducing the temporary pixels’ number will be discussed next.

A spot-generating hologram can be expressed as shown below:

\[ h(x, y) = \frac{e^{j \Phi_1} e^{-j(u_1 x + v_1 y)} + e^{j \Phi_2} e^{-j(u_2 x + v_2 y)} + \ldots + e^{j \Phi_i} e^{-j(u_i x + v_i y)}}{|e^{j \Phi_1} e^{-j(u_1 x + v_1 y)} + e^{j \Phi_2} e^{-j(u_2 x + v_2 y)} + \ldots + e^{j \Phi_i} e^{-j(u_i x + v_i y)}|^2} \]

where \( x \) and \( y \) are the coordinates on the hologram plane, \( u_i \) and \( v_i \) are the coordinates of the \( i \)th spot on the reconstruction plane, \( h(x, y) \) is the phase-only hologram, and \( \Phi_i \) is optimum phase of \( i \)th target spots.
According to Eq. (2), in order to generate $S$-target spots, at least $S$-temporary pixels are needed. The OPPO algorithm requests choosing the pixels randomly in order to ensure that both the high and low frequencies are represented. This criterion was preserved in the improved algorithm.

OPPO also needs the temporary pixels to be 4 to 5 times greater than the target spots. This principle is used to ensure fairness between spots and fair representation of the hologram's surface, especially when there are two or a small number of target spots. But, in our research, the hologram efficiency is the more important factor that needs to be considered.

It was found that the generated holograms may have a low efficiency when creating regular positioned target spots using $S$-temporary pixels in simulation. To avoid this situation, a judgment criterion was added to the program as the step 5 in Sec. 2.

The cut-off value of the hologram efficiency, $\eta$, is set to determine whether recalculations are needed once the resulting hologram is produced. The program reselects another group of temporary pixels randomly and recalculates the hologram until its efficiency reaches a satisfactory value (e.g., 80%). After many simulations, we found that the hologram only needed to be recalculated once, at most, in order to achieve a high efficiency.

It should be noted that if the target spots are irregularly (or randomly) positioned, the hologram efficiency keeps high and the recalculation step can be skipped. The judgment criterion is only used for ensuring high hologram efficiency, if necessary.

In the simulations, we calculated holograms with $256 \times 256$ pixels, and the number of temporary phase levels was 2. The number of the randomly positioned $S$-target spots was 1 to 10, so the efficiency judgment criteria did not actually work.

The hologram efficiency and the IMSE of the output spots when using $S$-temporary pixels are shown in Figs. 1(a) and 1(b) respectively.

Figure 1 shows that the hologram efficiency is still higher than 80% when using randomly chosen $S$-temporary pixels, and IMSE of the reconstruction is on the order of $10^{-4}$ and can be ignored too.

We conclude that decreasing the number of phase levels of a temporary hologram while reducing the number of temporary pixels results in a satisfactory hologram efficiency and the fairness between output spots.

### 3.3 Comparisons of the Computation Speeds

Both the original OPPO algorithm and the improved algorithm were written in the Matlab language for comparing their computation speeds.

In the calculations, the time required to flip and test a temporary pixel is $t_1$, and the fourth step of the algorithm, which consists of one FFT calculation, phase rounding operation, and some other computations, takes $t_0$. Therefore, the whole run time of the program will be

$$t_{\text{total}} = p \times S_\ell \times t_1 + t_0,$$

where $S_\ell$ is the number of the temporary pixels, $p$ is the number of the temporary hologram’s phase levels, and $S$-target spots will be created.

For OPPO, $S_\ell$ is 4S to 5S and $p$ is equal to the number of SLM phase levels, while for the improved algorithm $S_\ell$ is $S$ and $p$ is equal to 2. It is obvious that the improved algorithm greatly reduces the number of flip operations, which accelerates the computing speed.

The holograms calculated by the improved algorithm showed good performance in terms of hologram efficiency and fairness between output spots, as shown in Sec. 3.2.

A particular example of a 4-target-spot hologram with $256 \times 256$ pixels is shown in Fig. 2. The improved algorithm only required 0.8 s to achieve 80.91% efficiency. The original OPPO algorithm needed about 35 s to achieve the same efficiency using 16 temporary pixels and 256 phase levels.

### 4 Experiments

In the holographic optical tweezers platform, a Nd:YAG solid-state laser with 1 W power operating at 532 nm was used as the light source. The holograms generated by the improved algorithm were put into the BNS PM512 phase-only SLM ($512 \times 512$ pixels, 256 phase levels), and the
modulated laser beam was tightly focused using a 100 × NA1.25 Nikon oil-immersion microscope objective lens. Eight microparticles (made of polystyrene; diameter 2.5 μm) were trapped and manipulated.

Figure 3 depicts the optical tweezers system used in the experiment. A linearly polarized laser beam was expanded and collimated by a beam expander. Before illuminating the phase-only SLM, the polarization direction and incident angle of the beam were adjusted by a half-wave plate and mirrors M1 and M2 to meet the conditions specified by the manufacturer. The SLM modifies the phase of the incoming beam wavefront. A telescope was used to reduce the size of the modified beam to fill the entrance pupil of the objective lens and image the SLM onto the pupil plane. After passing through the telescope, the beam was reflected by a dichroic mirror, which was used to separate the laser beam and light of illuminator, and then focused into the sample plane using the high numerical aperture microscope objective lens. A CCD camera recorded the experiments.

In these experiments, the positions of the target spots were determined using a mouse. Every updated operation was accomplished within 1 s.

Figures 4(e)–4(g) are the recorded photos of eight trapped and controlled microparticles, and the corresponding 256-gray-level holograms obtained by the improved algorithm are shown in Figs. 4(a)–4(d).

5 Conclusions
By reducing the number of temporary phase levels to 2, choosing S (equal to the number of the target spots) temporary pixels with random positions, and using FFT, the calculation time required to generate a hologram was greatly reduced. The improved algorithm still performs well in terms of the hologram efficiency and fairness between the target spots. In addition, in order to avoid the low efficiency problem, the cut-off efficiency value was set to determine whether recalculations are needed. The experimental results demonstrate that real-time trap and manipulation of a small number of microparticles can be accomplished with the improved algorithm. By utilizing high-performance computers, the program’s run time can be further shortened, which could make the real-time trap and manipulation of a large number of micro-particles possible.

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References
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